Multivariable Calculus Common Topics List[1]

1. Multivariable Functions
2. 3-D space
   (a) Distance
   (b) Equations of planes, spheres, etc.
   (c) Two-variable function graphs
   (d) Sections, level curves, and contour diagrams
3. Vectors
   (a) Arithmetic on vectors, graphically and by components
   (b) Dot Product and projection
   (c) Cross Product
4. Limits are more complicated than in the one-variable case
5. Partial Derivatives
   (a) Compute using the de nition of partial derivatives
   (b) Compute using di erentiation rules
   (c) Approximate given a contour diagram or other info about a function
   (d) Estimate signs from real-world desciption
   (e) Find the tangent plane
   (f) Compute higher-order partials
   (g) Mixed partials are equal under certain conditions
6. Directional derivatives
   (a) Estimate from contour diagram
   (b) Compute using limit de nition
   (c) Compute using dot product with the gradient
7. Gradient
   (a) Compute the gradient
   (b) Points in the direction of fast3ah51
   (c) Length is the directional derivative in that direction
   (d) Perpendicular to level set
   (e) Draw gradient vector given a contour diagram

[1] This liah51was approved by the department on 4/10/19

8. Curl and Divergence
9. Chain Rule
10. Optimization
    (a) Locate and classify critical points in a contour diagram
    (b) Find critical points given a formula
    (c) Find maxima and minima
    (d) Second derivative test
    (e) Extreme value theorem, including understanding of closed and bounded
    (f) Lagrange multipliers
11. Integration
    (a) Predict the sign of a multiple integral
    (b) Compute a multiple integral
    (c) Sketch region of integration
    (d) Choose or change the order of integration
    (e) Polar and cylindrical coordinates
12. Parametrized Curves
    (a) Construct parametrizations of lines, circles, and explicitly defined curves
    (b) Velocity and speed
13. Vector Fields
    (a) Sketch a vector field with a given formula
    (b) Recognize a conservative (gradient) vector field
    (c) Find a formula for a potential function of a vector field
14. Line Integrals
    (a) Given a picture of a vector field, predict the sign of a line integral
    (b) Compute a line integral using explicit parametrization formula
    (c) Compute arc length of a curve
    (d) For a gradient field, compute using the fundamental theorem of line integrals
    (e) For a gradient field, compute using a reparametrization and path independence
    (f) For a gradient field, the line integral over a loop is zero
15. Green's Theorem

Suggested Additional Topics

Paraboloids, hyperboloids, and ellipsoids

Spherical Coordinates

Find plane tangent to implicit surface by viewing the surface as a level set

Implicit differentiation

Jacobian formula for reparametrizing a multiple integral into a different coordinate system

Surface Integrals over planes, spheres, cylinders

Stokes's Theorem

Divergence Theorem

Optional Topics

Level Surfaces

- limit proofs

Differentiability is more complicated than the one-variable case

Optimization on the boundary of a region by substitution

Jacobian formula for surface integral over an arbitrary surface

Proofs using vectors

Signs of second partials from contour diagram

Use limits to discuss existence of global maxima and minima

Intersections of a curve with a surface

Collisions and intersections of parametrized curves

Distances involving parametrized lines

Parametrize a complicated curve by summing parametrizations of simple component motions